

# Some notes on calculating GUP/NCBHs in LXDs

Sven Köppel

koeppel@fias.uni-frankfurt.de

Institut für theoretische Physik  
Frankfurt Institute for Advanced Sciences  
Goethe-Universität Frankfurt

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Calc-15b

## The Fourier integral, 3 dimensions

$$\begin{aligned}
 \int d^3p f(p) e^{i\vec{x}\cdot\vec{p}} &= \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta \int_0^\infty dp p^2 f(p) e^{i\vec{x}\cdot\vec{p}} \\
 &= 2\pi \int_{-1}^{+1} d\cos\theta \int_0^\infty dp p^2 f(p) e^{ixp \cos\theta} \\
 &= 2\pi \int_0^\infty \frac{1}{ixp} dp p^2 f(p) \left( e^{+ixp} - e^{-ixp} \right)
 \end{aligned}$$

## The 3d Fourier integral $\rightarrow$ effective 1d

$$\begin{aligned}
 \dots &= \frac{2\pi}{ix} \int_0^\infty dp \, p \, f(p) \left( e^{+ixp} \quad \color{green}{=} \quad e^{-ixp} \right) \\
 &= \frac{2\pi}{ix} \left( \int_0^\infty dp \, p \, f(p) e^{+ixp} \quad \color{green}{=} \quad \int_0^\infty dp \, p \, f(p) e^{-ixp} \right) \\
 &= \frac{2\pi}{ix} \left( \int_0^\infty dp \, p \, f(-p) e^{+ixp} \quad \color{yellow}{+} \quad \underbrace{\int_{-\infty}^0 dp' \, p' \, f(\color{green}{-}p') e^{+ixp'}}_{\substack{\int_a^b = \color{green}{-} \int_b^a \quad \text{and} \quad p' = \color{green}{-}p}} \right) \\
 &= \frac{2\pi}{ix} \int_{\color{green}{-\infty}}^{\color{yellow}{+\infty}} dp \, p \, \underbrace{\left[ f(p)\Theta(p) + f(-p)\Theta(-p) \right]}_{=f(|p|)} e^{+ixp}
 \end{aligned}$$

And  $\int dp \, p \, f(|p|) e^{ixp} \in \mathbb{C} \setminus \mathbb{R} \quad \Rightarrow \quad i \int \dots \in \mathbb{R} \quad \checkmark$

Does the same hold also in  $3 + n$  dimensions?  $n \in \mathbb{N}_0$

$$\begin{aligned} \int d^{3+n} f(p) e^{i\vec{x}\vec{p}} &\propto \int_0^\infty \frac{1}{ixp} dp p^{2+n} f(p) \left( e^{+ixp} - e^{-ixp} \right) \\ &\propto \int_{-\infty}^\infty dp p^{1+n} \underbrace{\left[ f(p)\Theta(p) + (-1)^n f(-p)\Theta(-p) \right]}_{:=v(p)} e^{+ixp} \end{aligned}$$

Spherical coordinates: ✓

Effective 1d notation: ✓

Issue with Holomorphy? Perhaps

Real result? ✓

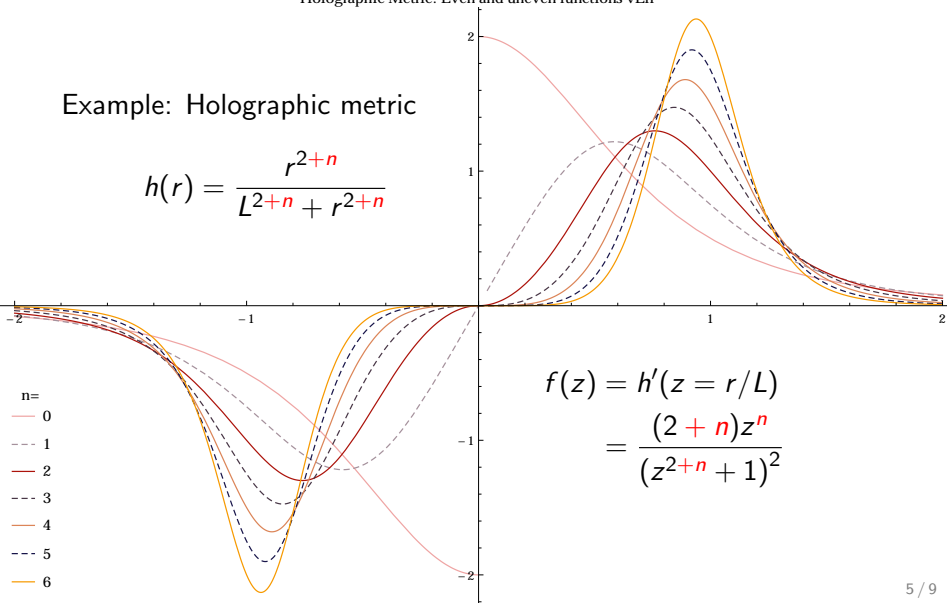
- Even  $n = 0, 2, 4$ :  $v(p) = p^{\text{odd}} f(|p|) \Rightarrow f \in \mathbb{C} \setminus \mathbb{R} \Rightarrow i f \in \mathbb{R}$
- Odd  $n = 1, 3, 5$ :  $v(p) = p^{\text{even}} [\text{odd}] \Rightarrow f \in \mathbb{C} \setminus \mathbb{R} \Rightarrow i f \in \mathbb{R}$

# The $\Theta$ composition issue

Holographic Metric: Even and uneven functions vEff

Example: Holographic metric

$$h(r) = \frac{r^{2+n}}{L^{2+n} + r^{2+n}}$$



## Ignoring facts: Applying Cauchy theorem

All  $f(p)$  choices look roughly like

$$f(p) \approx \frac{1}{(1 + L \cdot p \cdot \bullet)^{\bullet}} = \frac{1}{(1 + q \cdot \bullet)^{\bullet}}$$

All ( $\bullet - 1$ ) Poles are on the unit circle ( $1 = e^{i\varphi}$ ):

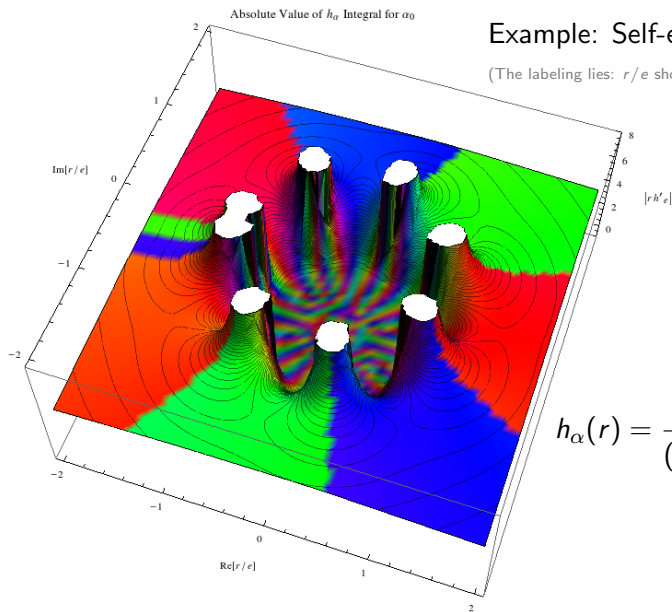
$$q = (-1)^{\frac{1}{\bullet}} = \exp \left\{ \frac{i\pi + 2\pi ik}{\bullet} \right\} \forall k \in \mathbb{N}$$

$\Rightarrow$  It's simple to sketch the setting!

## From the complex plane...

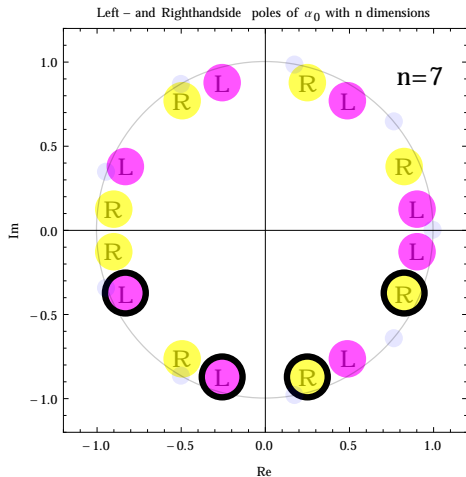
Example: Self-encoding metric

(The labeling lies:  $r/e$  should be  $r/L$ )



$$h_\alpha(r) = \frac{r^{3+n}}{(r^\alpha + L^\alpha/2)^{3+n/\alpha}}$$

...to the pole diagram



## Complex Points

- L Poles of  $f(-p)$
- R Poles of  $f(p)$
- Chosen poles
- Roots of unity

Only work left:

$$\int dz = \sum \text{Res}_{z \rightarrow z_0}$$



## Results: Work must be checked

- Correct for no extra dimensions ( $n = 0$ ).
- Complex values do not vanish  $\rightarrow$  Complex masses.