The Generalized Uncertainty Principle in extradimensions

Sven Köppel, Marco Knipfer, Maximiliano Isi, Jonas Mureika, Piero Nicolini

LMU|LA
Loyola Marymount University

FIAS Frankfurt Institute for Advanced Studies

Caltech

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

KSM 2015 • Student session • 21. Jul 2015, 14:40-15:00

The Generalized Uncertainty Principle

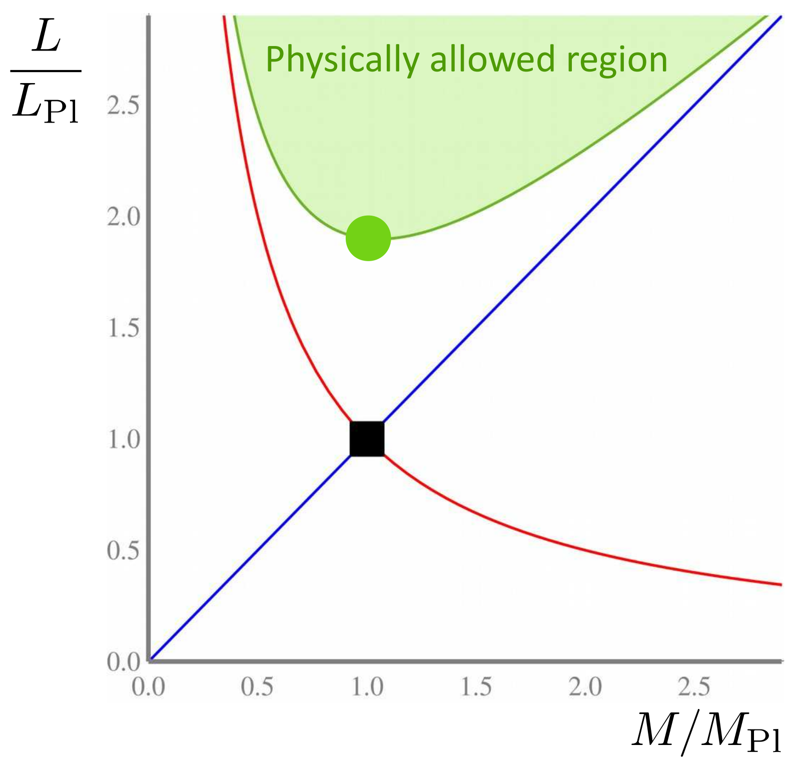
1

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+f(p)\right)$$

$$f(p) = \beta p^2$$

Veneziano 1986
Amati, Veneziano, Ciafaloni 1995
Michele Maggiore 1995
Kempf, Mangano, Mann 1996

UV-Complete Gravity,
Minimal length



Aurilia, Spallucci 2002
Dvali, Gomez 2010
Carr 2014

GUP: Hot remnants

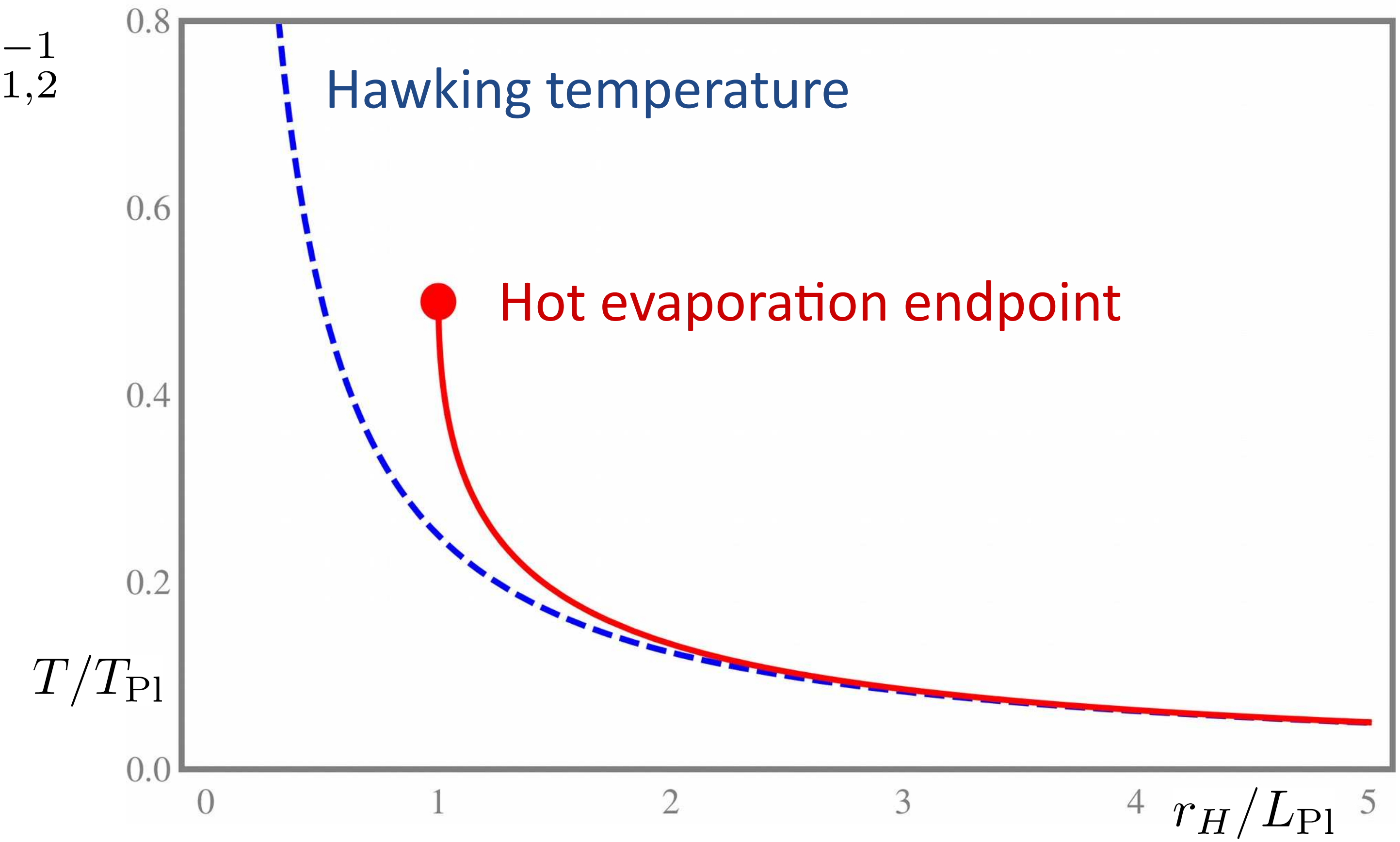
2

$$\Delta x \Delta p \sim 1 + (L_{\text{Pl}} \Delta p)^2$$

Adler, Santiago 1999
Adler, Chen, Santiago 2001

$$\Delta p_{1,2} \sim \frac{\Delta x}{2L_{\text{Pl}}^2}\left(1 \pm \sqrt{1 - \frac{L_{\text{Pl}}^2}{\Delta x^2}}\right)$$

$$T \sim M^{-1} \sim \Delta p_{1,2}^{-1}$$



GUP inspired Black holes

Hilbert space representation

$$\int \frac{\mathrm{d}^3 p}{1 + f(p)} \; |p\rangle\langle p| = 1$$

Kempf, Mangano, Mann 1996

Approximation in nonlocal Lagrangian gives nonlocal EFE

$$\mathcal{A}(\Box) G_{\mu\nu} = 8\pi G \; T_{\mu\nu}$$

Choose $\mathcal{A}(\Box) = \dfrac{1}{1 + \beta\Box}$

Krasnikov 1987, Modesto 2012, 2013 Moffat 2012

What is the effect on a Schwarzschild BH?

$$T^0_0 = -M\delta^{(3)}(r)$$

Isi, Mureika, Nicolini 2013 Balazin, Nachbagauer 1994

Modified Newton's potential

$$\mathrm{d}s^2 = -\left(1 - \frac{2G\mathcal{M}(r)}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$\mathcal{M}(r) = -4\pi M \int_0^r \mathrm{d}r' \; r'^2 \; \mathcal{A}(\Box)\delta^3(r') \sim \int_0^r \mathrm{d}r' \; r'^2 \int \frac{\mathrm{d}^3 p}{1 + \beta\vec{p}^{\,2}} e^{i\vec{x}\cdot\vec{p}}$$

Analytical result:

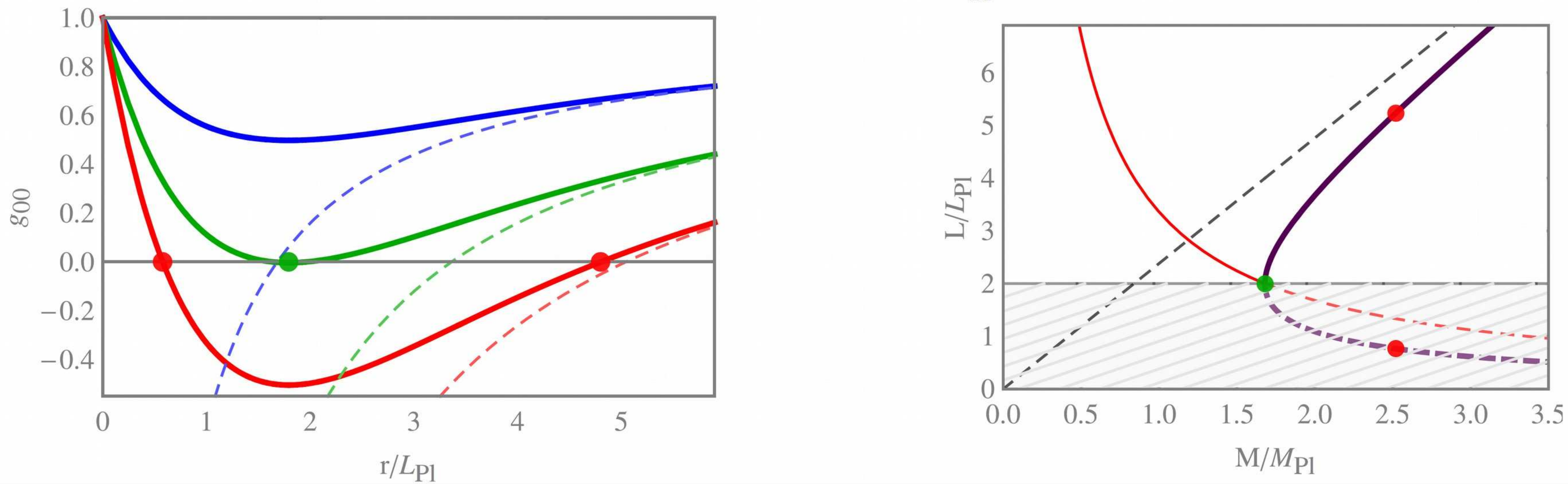
$$\mathcal{M}(r) = M\left(1 - e^{-r/\sqrt{\beta}} - (r/\sqrt{\beta})e^{-r/\sqrt{\beta}}\right)$$

GUP inspired Black holes: Self completeness

$$\mathrm{d}s^2 = -\left(1 - \frac{2GM}{r}\gamma(2, r/\sqrt{\beta})\right)\mathrm{d}t^2 + \left(1 - \frac{2GM}{r}\gamma(2, r/\sqrt{\beta})\right)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Isi, Mureika, Nicolini 2013

M=0.84, 1.68, 2.52M_{Pl}



Self complete: Can relate free parameter to fundamental length

$$\beta = 1.534 L_{\mathrm{Pl}}$$

The Kempf-Mangano-Mann model in N+1 dimensions

Apply the momentum space regularization in *N* spatial dimensions (ADD model)

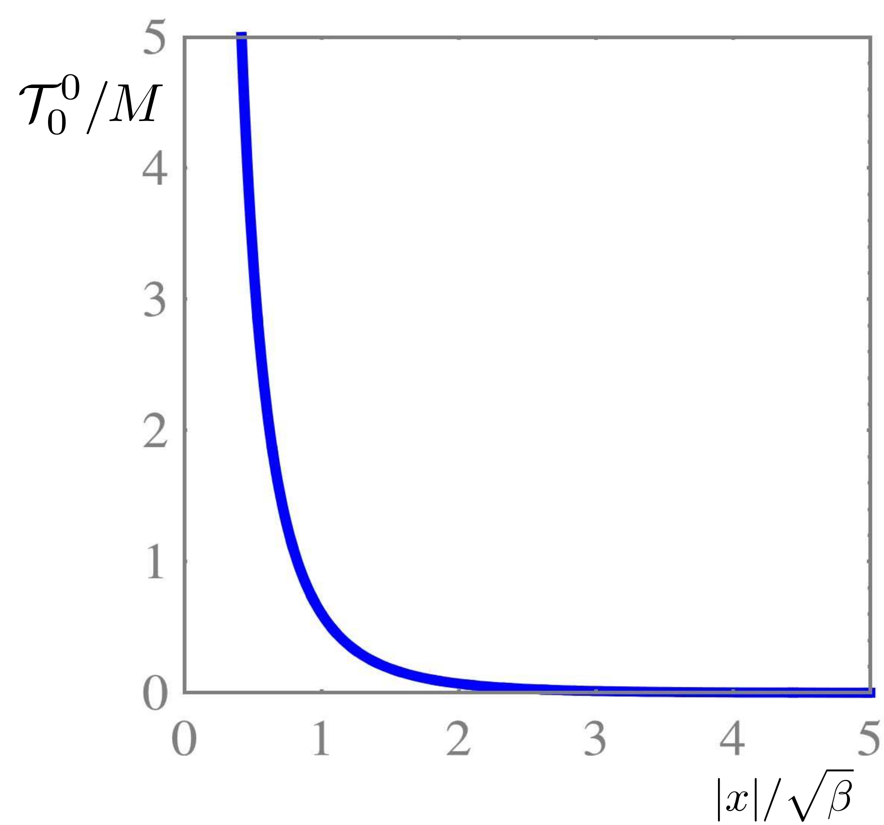
$$\int \frac{\mathrm{d}^N p}{1+\beta \vec{p}^{\,2}} \, |p\rangle\langle p| = 1$$

Kempf, Mangano, Mann 1996
Scardigli, Casadio 2003

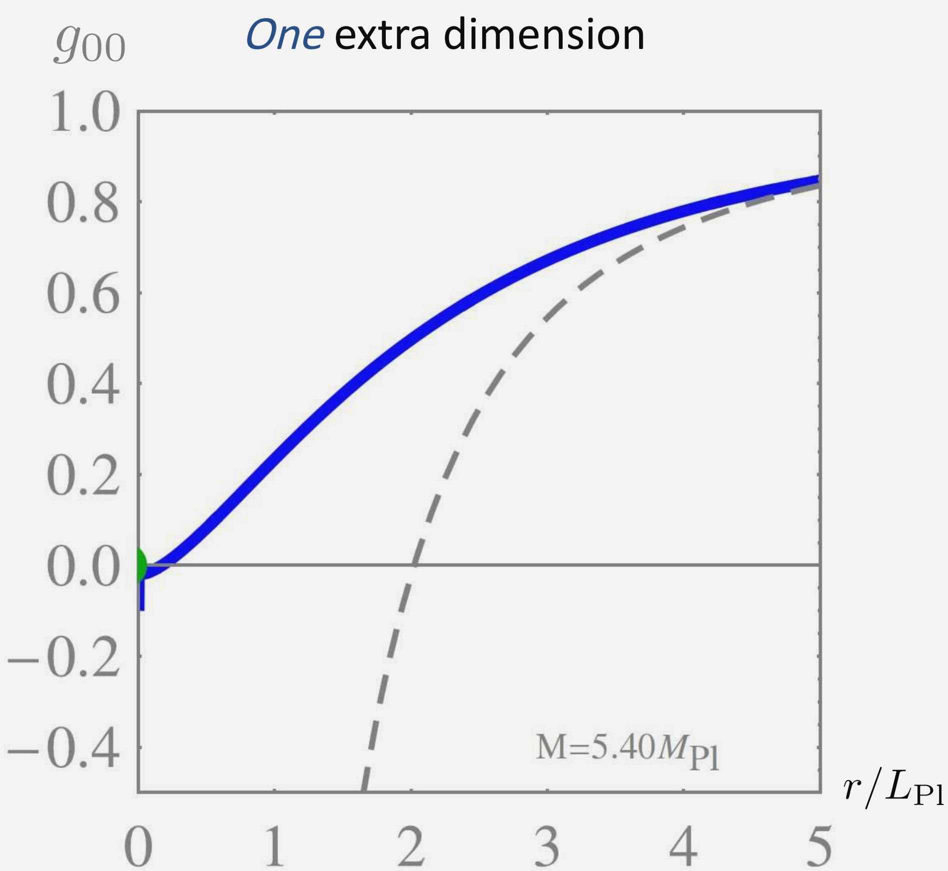
We can solve the nonlocal operator analytically in any number of extradimensions

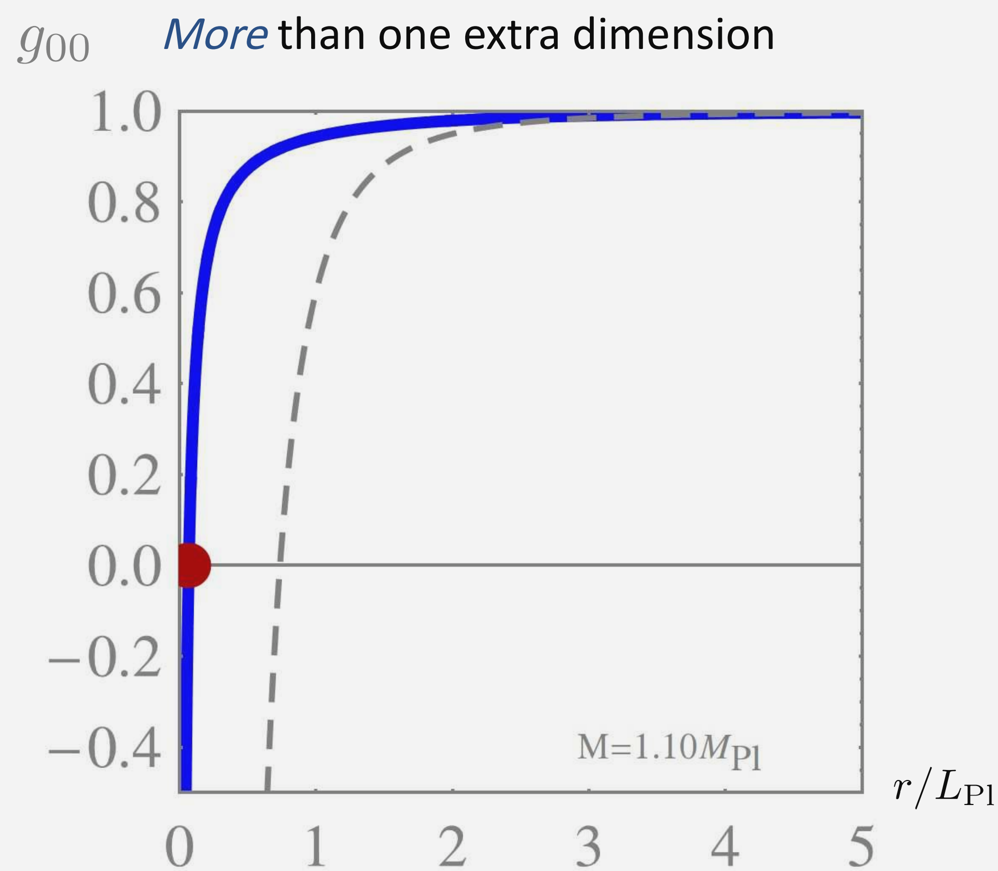
$$\mathcal{T}_0^0 = \frac{M}{(2\pi)^N} \int \frac{\mathrm{d}^N p \; e^{i\vec{x}\cdot\vec{p}}}{1+\beta p^2}$$

$$\sim \frac{1}{|x|^{N/2-1}} K_{N/2-1}\left(|x|/\sqrt{\beta}\right)$$



GUP-inspired BH in higher dimensions





No more remnants in higher ($N > 4$) dimensions.

$$\mathcal{T}_0^0 \sim M \int \frac{\mathrm{d}^N p \; e^{i\vec{x}\cdot\vec{p}}}{1+\beta p^2} \sim M \int \mathrm{d}p \; e^{irp} \; p^{N-3}$$

Retracing a popular Gedankenexperiment

The Heisenberg microscope in n LXDs:

Adler 2010
Carr 2014

$$\Delta x \sim \Delta x_c + \Delta x_g$$

$$\Delta x_c \sim \lambda \sim 1/\Delta p$$

$$\Delta x_g \sim G_{(n)} \frac{M_{\text{eff}}}{r^{2+n}} \frac{r^2}{c^2} \sim G_{(n)} \frac{\Delta p}{r^n} \sim L_{(\mathrm{n})}^{2+n} \Delta p^{1+n}$$

This Gedankenexperiment motivates a modified GUP:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+(\sqrt{\beta}p)^{2+n}\right)$$

$$\mathcal{T}_0^0 \sim M \int \frac{\mathrm{d}^{3+n}p \; e^{i\vec{x}\cdot\vec{p}}}{1+(\sqrt{\beta}p)^{2+n}} \sim M \int \mathrm{d}^N p \; e^{i\vec{x}\cdot\vec{p}}$$

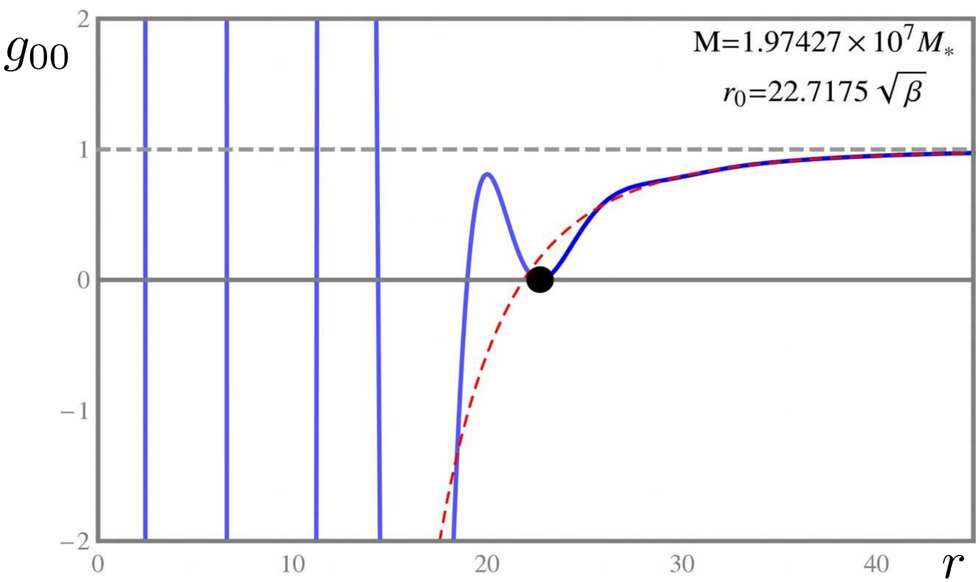
Constant integrand: Sounds reasonable

We can solve this spacetime analytically in any dimension.

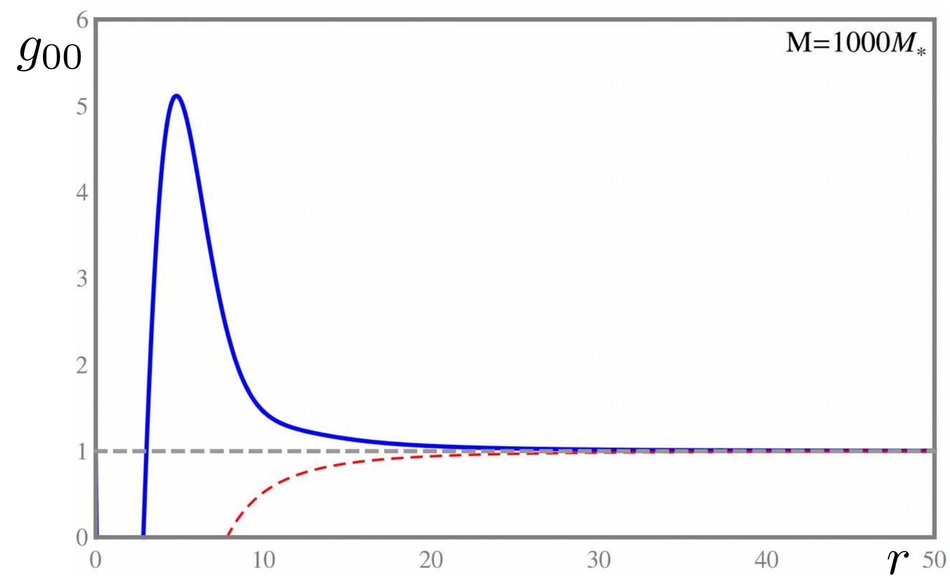
Modified GUP inspired Black Holes

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+(\sqrt{\beta}p)^{2+n}\right)$$

Gravitational potential with 4 extra dimensions:



Gravitational potential with 1 extra dimension:



Pathologies!

Conclusion

1. GUP in 4d works well,
   Our metric solves the hot remnant issues by Adler

2. Higher dimensions are different.

- Improve GUP. *Example:*

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+(\sqrt{\beta}p)^{(2+n)/(1+n)}\right)$$

Maziashvili 2013

- Improve NLG.
  Include higher derivative terms in nonlocal operator

- Propose nonsingular model.
  Model engineering

Hayward 2006

The Generalized Uncertainty Principle in extradimensions

Sven Köppel, Marco Knipfer, Maximiliano Isi, Jonas Mureika, Piero Nicolini

GOETHE UNIVERSITÄT FRANKFURT AM MAIN
Caltech
LMU|LA Loyola Marymount University
FIAS Frankfurt Institute for Advanced Studies

Thank you for your attention.

Ongoing

Köppel, Isi, Knipfer, Mureika, Nicolini
Self-Completeness and the GUP in extradimensions
Köppel, Knipfer, Dirkes, Frassino, Nicolini, Bleicher
GUP and BHs – a pedagogical review

References

Isi, Mureika, Nicolini
Self-Completeness and the GUP [JHEP 2013]

Ways out

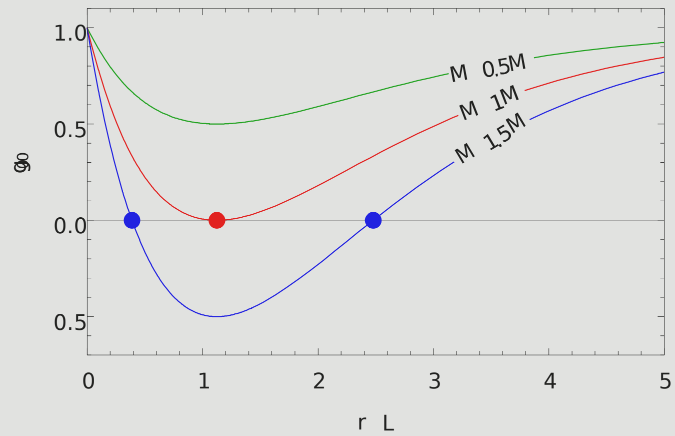
One option: Black hole engineering to mimic 4d in any number of extra dimensions

1. Postulate the same energy density in any D

$$\rho(r) \sim \frac{1}{r}\exp(-r/L_*)$$

2. Find the generating nonlocal operator

$$\mathcal{A}^{-2}(p) = \frac{1}{M}\int \rho(r)e^{-i\vec{x}\cdot\vec{p}}\mathrm{d}^N x$$
$$\sim \frac{i}{p}\left(\frac{1}{(1+ip)^{N-2}} - \frac{1}{(1-ip)^{N-2}}\right)$$



Or: Search for *other* GUP modifications. Example:

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \beta \hbar^{(\alpha-1)/\alpha} G_N^{1/\alpha(n+1)} \Delta P^{(n+2)/\alpha(n+1)}$$

Maziashvili 2013

Can produce an attractive extension

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + (\sqrt{\beta}p)^{(2+n)/(1+n)}\right)$$

which is porbably related to BHs in spectral number of dimensions (Unparticles). *Metric not solved yet.*