

# The Generalized Uncertainty Principle in extradimensions

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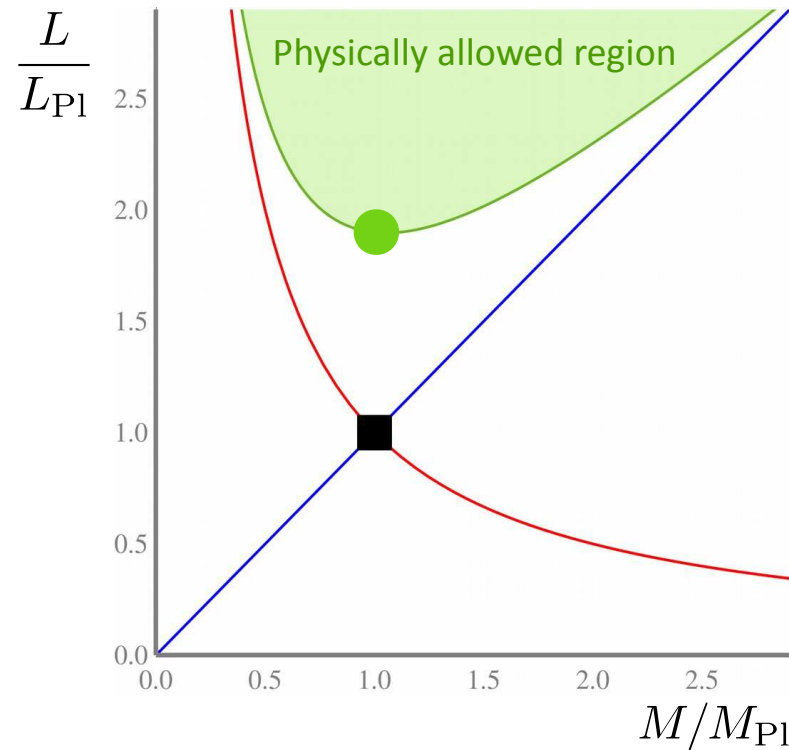
KSM 2015 • Student session • 21. Jul 2015, 14:40-15:00

# The Generalized Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2} (1 + f(p)) \quad f(p) = \beta p^2$$

Veneziano 1986  
 Amati, Veneziano, Ciafaloni 1995  
 Michele Maggiore 1995  
 Kempf, Mangano, Mann 1996

UV-Complete Gravity,  
 Minimal length



Aurilia, Spallucci 2002  
 Dvali, Gomez 2010  
 Carr 2014

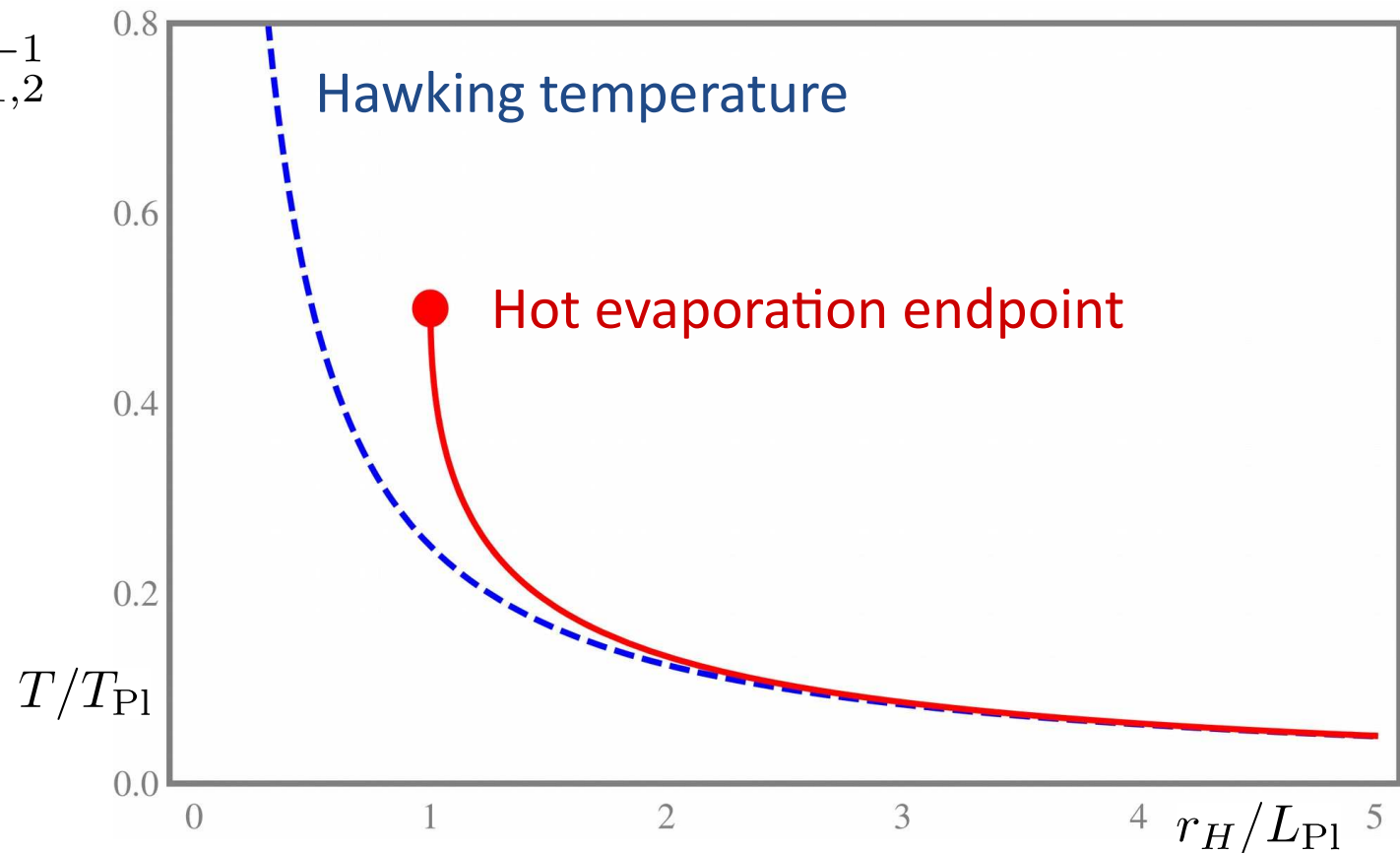
# GUP: Hot remnants

$$\Delta x \Delta p \sim 1 + (L_{\text{Pl}} \Delta p)^2$$

$$\Delta p_{1,2} \sim \frac{\Delta x}{2L_{\text{Pl}}^2} \left( 1 \pm \sqrt{1 - \frac{L_{\text{Pl}}^2}{\Delta x^2}} \right)$$

Adler, Santiago 1999  
Adler, Chen, Santiago 2001

$$T \sim M^{-1} \sim \Delta p_{1,2}^{-1}$$



# GUP inspired Black holes

Hilbert space representation

$$\int \frac{d^3 p}{1 + f(p)} |p\rangle \langle p| = 1$$

Kempf, Mangano, Mann 1996

Approximation in nonlocal Lagrangian gives nonlocal EFE

$$\mathcal{A}(\square) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\text{Choose } \mathcal{A}(\square) = \frac{1}{1 + \beta \square}$$

Krasnikov 1987, Modesto 2012, 2013 Moffat 2012

What is the effect on a Schwarzschild BH?

$$T_0^0 = -M \delta^{(3)}(r)$$

Isi, Mureika, Nicolini 2013 Balazin, Nachbagauer 1994

Modified Newton's potential

$$ds^2 = - \left( 1 - \frac{2G\mathcal{M}(r)}{r} \right) dt^2 + \left( 1 - \frac{2G\mathcal{M}(r)}{r} \right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\mathcal{M}(r) = -4\pi M \int_0^r dr' r'^2 \mathcal{A}(\square) \delta^3(r') \sim \int_0^r dr' r'^2 \int \frac{d^3 p}{1 + \beta \vec{p}^2} e^{i\vec{x}\cdot\vec{p}}$$

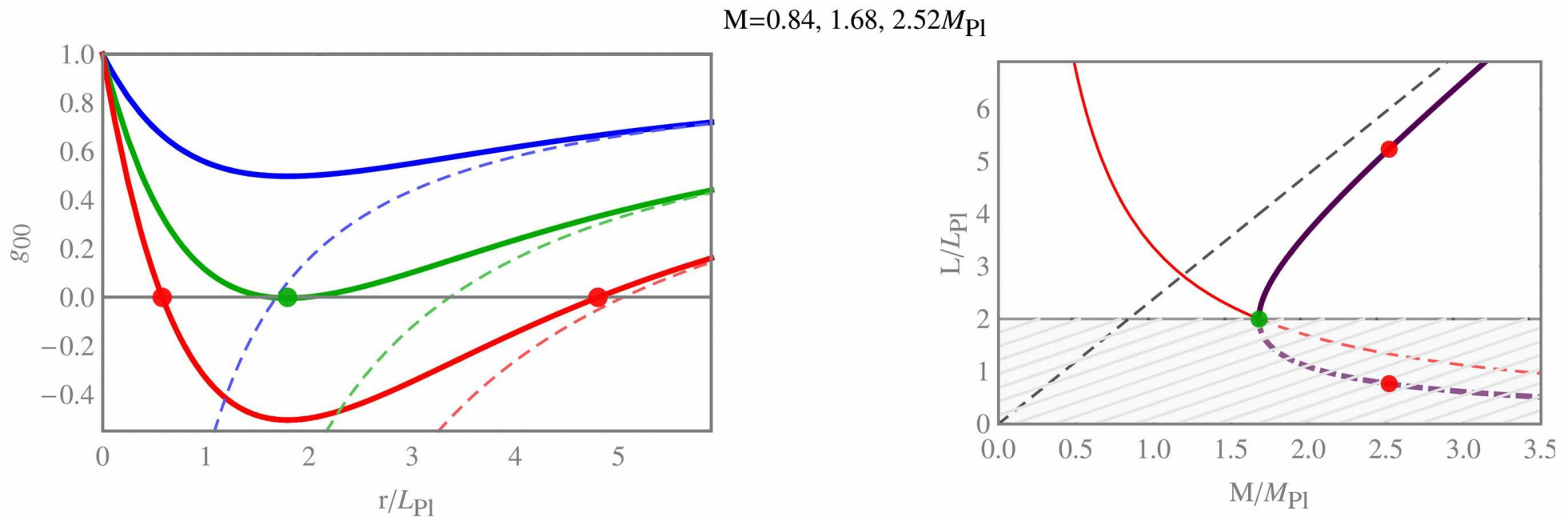
Analytical result:

$$\mathcal{M}(r) = M \left( 1 - e^{-r/\sqrt{\beta}} - (r/\sqrt{\beta}) e^{-r/\sqrt{\beta}} \right)$$

# GUP inspired Black holes: Self completeness

$$ds^2 = - \left( 1 - \frac{2GM}{r} \gamma(2, r/\sqrt{\beta}) \right) dt^2 + \left( 1 - \frac{2GM}{r} \gamma(2, r/\sqrt{\beta}) \right) dr^2 + r^2 d\Omega^2$$

Isi, Mureika,  
Nicolini 2013



Self complete: Can relate free parameter to fundamental length

$$\beta = 1.534 L_{\text{Pl}}$$

# The Kempf-Mangano-Mann model in N+1 dimensions

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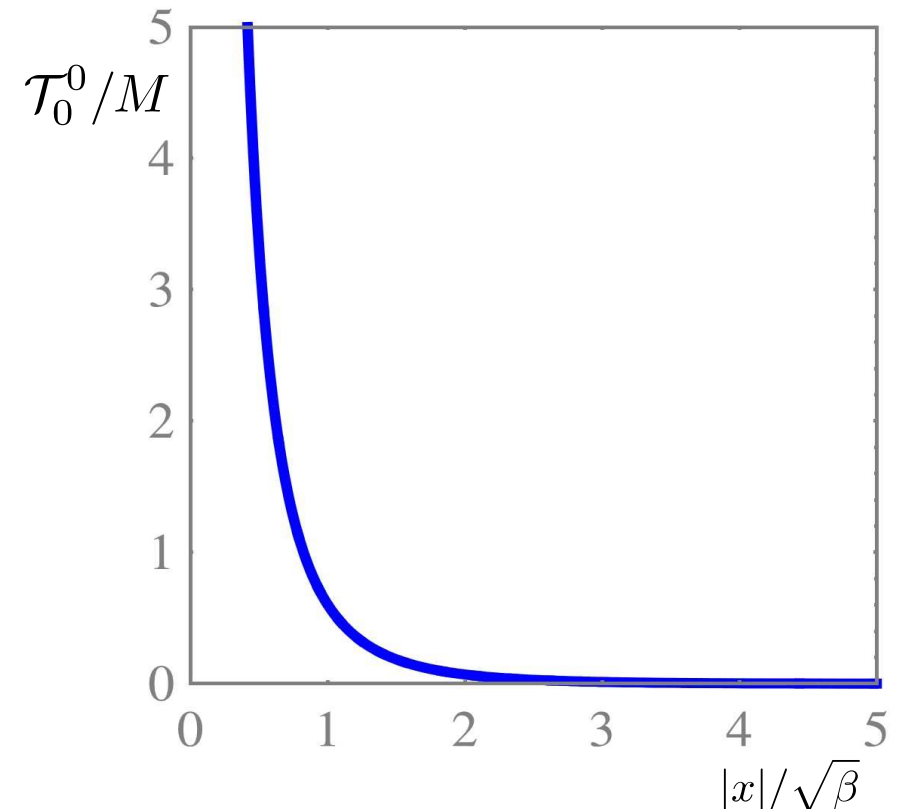
Apply the momentum space regularization in  $N$  spatial dimensions (ADD model)

$$\int \frac{d^N p}{1 + \beta \vec{p}^2} |p\rangle \langle p| = 1$$

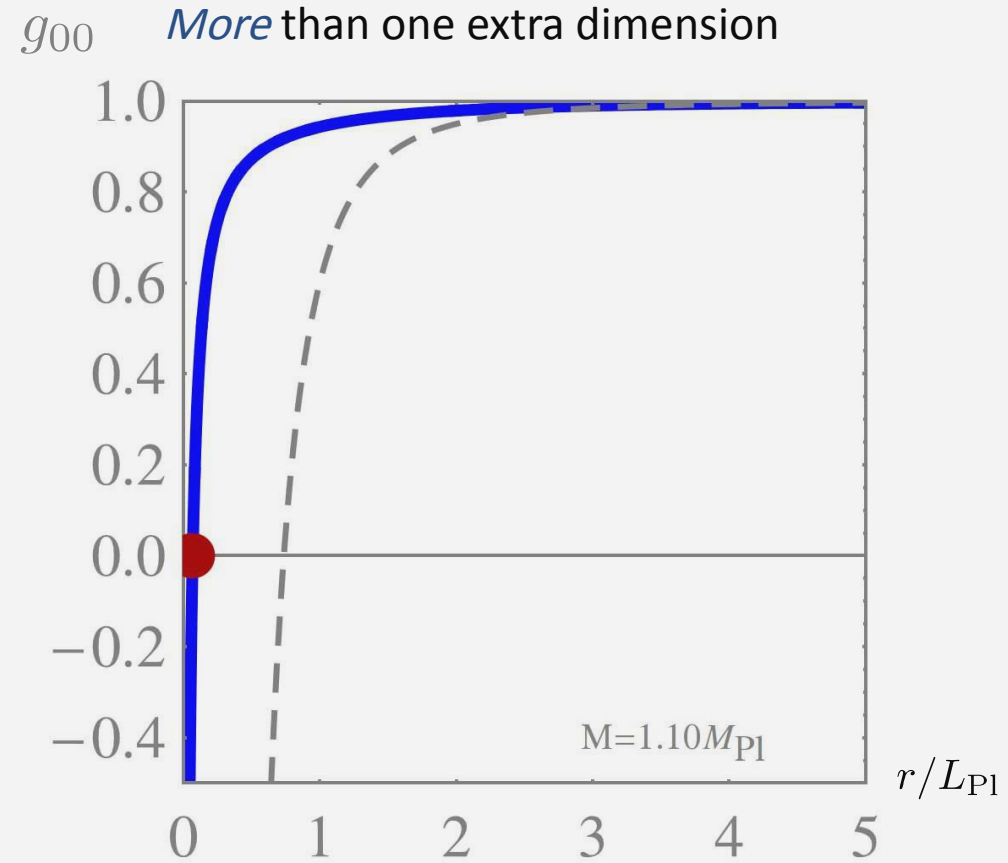
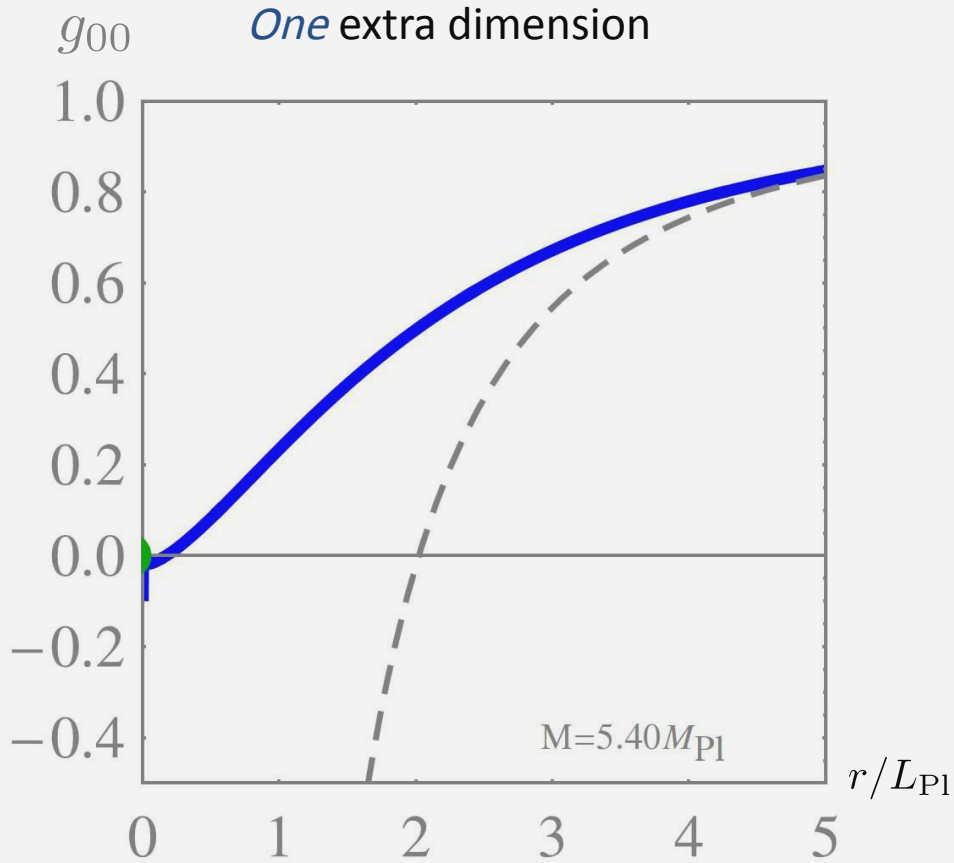
Kempf, Mangano, Mann 1996  
Scardigli, Casadio 2003

We can solve the **nonlocal operator analytically** in any number of extradimensions

$$\begin{aligned} \mathcal{T}_0^0 &= \frac{M}{(2\pi)^N} \int \frac{d^N p e^{i\vec{x}\cdot\vec{p}}}{1 + \beta p^2} \\ &\sim \frac{1}{|x|^{N/2-1}} K_{N/2-1} \left( |x|/\sqrt{\beta} \right) \end{aligned}$$



# GUP-inspired BH in higher dimensions



No more remnants in higher ( $N > 4$ ) dimensions.

$$\mathcal{T}_0^0 \sim M \int \frac{d^N p e^{i\vec{x}\cdot\vec{p}}}{1 + \beta p^2} \sim M \int dp e^{irp} p^{N-3}$$

# Retracing a popular Gedankenexperiment

The Heisenberg microscope in  $n$  LXDs:

$$\Delta x \sim \Delta x_c + \Delta x_g$$

$$\Delta x_c \sim \lambda \sim 1/\Delta p$$

$$\Delta x_g \sim G_{(n)} \frac{M_{\text{eff}} r^2}{r^{2+n} c^2} \sim G_{(n)} \frac{\Delta p}{r^n} \sim L_{(n)}^{2+n} \Delta p^{1+n}$$

Adler 2010  
Carr 2014

This Gedankenexperiment motivates a modified GUP:

$$\Delta x \Delta p \geq \frac{\hbar}{2} (1 + (\sqrt{\beta} p)^{2+n}) \quad \mathcal{T}_0^0 \sim M \int \frac{d^{3+n} p e^{i\vec{x} \cdot \vec{p}}}{1 + (\sqrt{\beta} p)^{2+n}} \sim M \int d^N p e^{i\vec{x} \cdot \vec{p}}$$

Constant integrand: Sounds reasonable

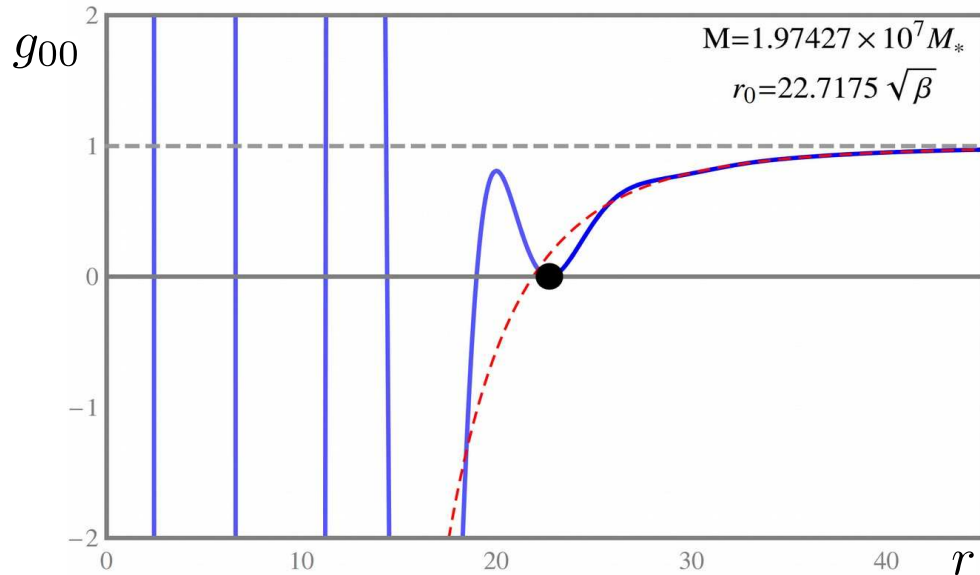
We can [solve](#) this spacetime [analytically](#) in any dimension.



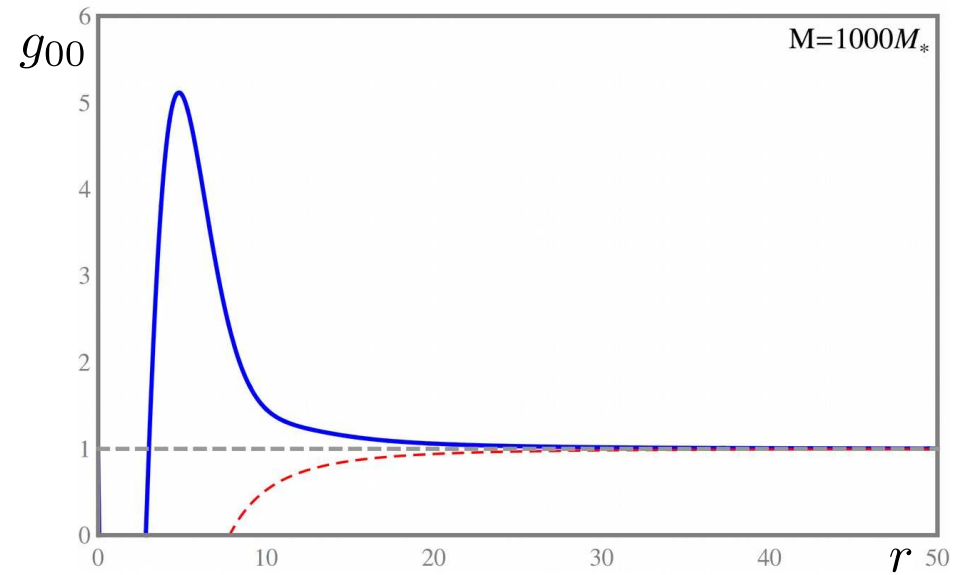
# Modified GUP inspired Black Holes

$$\Delta x \Delta p \geq \frac{\hbar}{2} (1 + (\sqrt{\beta} p)^{2+n})$$

Gravitational potential with 4 extra dimensions:



Gravitational potential with 1 extra dimension:



Pathologies!

# Conclusion

1. GUP in 4d works well,  
Our metric solves the hot remnant issues by Adler
2. Higher dimensions are different.

- Improve GUP. *Example:*

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left( 1 + (\sqrt{\beta} p)^{(2+n)/(1+n)} \right)$$

Maziashvili 2013

- Improve NLG.  
Include **higher derivative terms** in nonlocal operator

- Propose nonsingular model.  
Model engineering

Hayward 2006

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*Thank you for your attention.*

## *References*

Isi, Mureika, Nicolini  
Self-Completeness and the GUP  
[JHEP 2013]

## *Ongoing*

Köppel, Isi, Knipfer, Mureika, Nicolini  
Self-Completeness and the GUP  
in extradimensions

Köppel, Knipfer, Dirkes, Frassino, Nicolini, Bleicher  
GUP and BHs – a pedagogical review

# Ways out



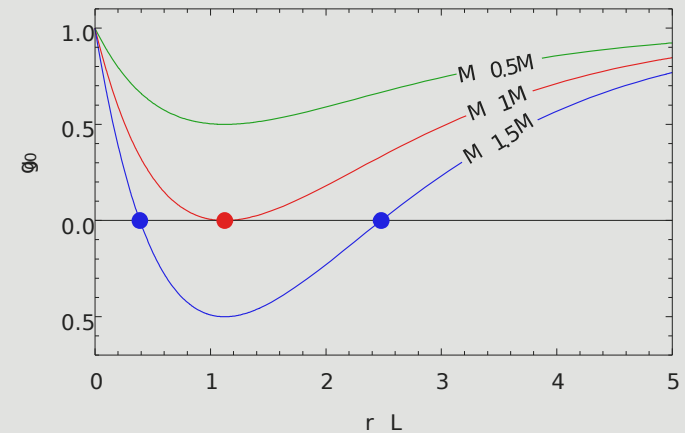
**One option:** Black hole engineering to mimic 4d in any number of extra dimensions

1. Postulate the same energy density in any D

$$\rho(r) \sim \frac{1}{r} \exp(-r/L_*)$$

2. Find the generating nonlocal operator

$$\begin{aligned} \mathcal{A}^{-2}(p) &= \frac{1}{M} \int \rho(r) e^{-i\vec{x}\cdot\vec{p}} d^N x \\ &\sim \frac{i}{p} \left( \frac{1}{(1+ip)^{N-2}} - \frac{1}{(1-ip)^{N-2}} \right) \end{aligned}$$



**Or:** Search for *other* GUP modifications. Example:

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \beta \hbar^{(\alpha-1)/\alpha} G_N^{1/\alpha(n+1)} \Delta P^{(n+2)/\alpha(n+1)}$$

Maziashvili 2013

Can produce an attractive extension

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left( 1 + (\sqrt{\beta p})^{(2+n)/(1+n)} \right)$$

which is probably related to BHs in spectral number of dimensions (Unparticles). *Metric not solved yet.*