Frolov: Mass-gap for black hole formation in **higher derivative** and ghost free **gravity**

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Sven Köppel
koeppel@fias.uni-frankfurt.de

Frankfurt Institute for Advanced Studies
Institut für theoretische Physik
Goethe-Universität Frankfurt

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Biswas 2013
Higher Derivative Gravity

In this section

Presenting the more readable results from Biswas, Gerwick, Koivisto, Mazumdar (arXiv:1110.5249, arXiv:1302.0532)

Linearized gravity $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The most general quadratic action:

$$S_q = \int d^4x \sqrt{-g}\, R_{\mu_1\nu_1\lambda_1\sigma_1} \mathcal{D}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2} R^{\mu_2\nu_2\lambda_2\sigma_2}$$

with $\mathcal{D} \supset \{\eta_{\mu\nu}, \nabla_\mu\}$.

Biswas 2013
Confining the Lagrangian: 14 Terms

$$
\begin{aligned}
S_q \;=\; & \int d^4x\sqrt{-g}\Big[RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\mu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R_\mu^\nu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
+ \; & R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R_\lambda^\rho F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} + R_\mu^{\nu_1\rho_1\sigma_1}F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}\Big] .
\end{aligned}
$$

Confining the Lagrangian: 14 Terms $\to$ 3 Terms

- Using the antisymmetric properties of the Riemann tensor,

$$R_{(\mu\nu)\rho\sigma} = R_{\mu\nu(\rho\sigma)} = 0$$

- and the Jacobi identity

$$\nabla_\alpha R_{\mu\nu\beta\gamma} + \nabla_\gamma R_{\mu\nu\beta\alpha} + \nabla_\beta R_{\mu\nu\gamma\alpha}$$

  Making 8 terms in GR.

- Linearizing: ∇_μ commute on Minkowski throwing out 3 terms due to symmetry

Biswas 2013

Confining the Lagrangian: 3 Terms

Linearized Action:

$$S_q = \int d^4x \left[R F_1(\Box) R + R_{\mu\nu} F_3(\Box) R^{\mu\nu} + R_{\mu\nu\lambda\sigma} F_{10}(\Box) R^{\mu\nu\lambda\sigma} \right]$$

can be casted in terms of $h_{\mu\nu}$ by expressions like

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]}), \ldots, R = \partial_\nu\partial_\mu h^{\mu\nu} - \Box h$$

yielding eq. (1) from the Frolov paper:

$$S_q = -\int d^4x \Big[\frac{1}{2} h_{\mu\nu} \Box a(\Box) h^{\mu\nu} + h_\mu^\sigma b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + h c(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} h \Box d(\Box) h + h^{\lambda\sigma} \frac{f(\Box)}{\Box} \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \Big]$$

Applications to special cases

The most general action

$$S_q = -\int d^4x \Big[\frac{1}{2} h_{\mu\nu} \Box a(\Box) h^{\mu\nu} + h^{\sigma}_{\mu} b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + h c(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} h \Box d(\Box) h + h^{\lambda\sigma} \frac{f(\Box)}{\Box} \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \Big]$$

GR Field equations can be used to show

$$b = 0, \quad c + d = 0, \quad b + c + f = 0,$$

with only two independent functions.

General Realtivity (GR)

Recover GR in infrared domain

$$S_q = -\int d^4x \Big[\frac{1}{2} h_{\mu\nu} \Box a(\Box) h^{\mu\nu} + h_\mu^\sigma b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + h c(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} h \Box d(\Box) h + h^{\lambda\sigma} \frac{f(\Box)}{\Box} \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \Big]$$

with $a(0) = c(0) = -b(0) = -d(0) = 1$ gives $\mathcal{L} = R$

Gauss-Bonnet (GB)

- Gauss-Bonnet term $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$
- is a special case with $a = c = 1$, so that $\mathcal{L} = R + \alpha(\Box)\mathcal{G}$.
- (Interesting only in $D \geq 4 + 1$ dimensions).

L(R) gravity

Also called $f(R)$ gravity: Replace R in Einstein-Hilbert action with $f(R)$, or:

$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R + \dots$$

with $a = 1, c = 1 - \mathcal{L}''(\Box)$.

Weyl gravity

- Using the Weyl tensor (traveless component of the Riemann tensor).
- Weyl tensor $C_{\mu\nu\alpha\beta} = 0$ when metric is (locally) conformally flat.
- Weyl-squared theory with $1/\mu$ suppression:

$$\mathcal{L} = R - \frac{1}{\mu^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$$

can be constructed with $a = 1 - \mu^{-2}\Box, c = 1 - \frac{1}{3}\mu^{-2}\Box$

Higher derivative (HD) gravity

$$a = \prod_{i=1}^{n} (1 - \mu_i^{-2} \Box)$$

$$c = \prod_{k=1}^{n_c} (1 - \nu_k^{-2} \Box)$$

i.e. a (in)finite amount of derivatives associated with energy scales.

Ghost free (GF) gravity

$$a = c = exp(-\Box/\mu^2)$$

equal to NCG (Noncommutative Geometry) / NCBH (Noncommuting Black Holes), and similar to GUP.

Static solutions of the linearized gravity equations

- Start with point-source Energy-momentum tensor

$$\tau_{\mu\nu} = \rho\delta^0_\mu\delta^0_\nu = m\delta^3(\vec{r})\delta^0_\mu\delta^0_\nu$$

- Compute metric in Newtonian limit

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)dx^2$$

by solving EFE (eg. geometric part:)

$$-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (a+b)\Box h^\mu_{\nu,\mu} + (c+d)\Box\partial_\nu h + (b+c+f)h^{\alpha\beta}_{,\alpha\beta\nu}$$

- Yields equations

$$2(a-3c)[\nabla^2\Phi - 4\nabla^2\Psi] = \kappa\rho$$
$$2(c-a)\nabla^2\Phi - 4c\nabla^2\Psi = -\kappa\rho\,.$$

"We seek functions $c(\Box)$ and $a(\Box)$, such that there are no ghosts and no $1/r$ divergence at short distances"

The solution

With $f = 0$ ($\Leftrightarrow$ non higher derivative theory),

$$4a(\nabla^2)\nabla^2\Phi = 4a(\nabla^2)\nabla^2\Psi = \kappa\rho = \kappa m \delta^3(\vec{r})$$

- For instance $a(\Box) = e^{-\Box/M^2}$ like in String theory
- Fourier transformation of eq ($\Uparrow$) gives

$$\Phi(r) \sim \frac{m}{M_p^2 r} \int \frac{dp}{p} e^{-p^2/M^2} \sin(p\, r) = \frac{m\pi}{2M_p^2\, r} \mathtt{erf}\left(\frac{rM}{2}\right)$$

- Error function $\lim_{r\to\infty} \mathtt{erf}(r) = 1$ is GR limit
- Error function $\lim_{r\to 0} \mathtt{erf}(r) = r$ makes Newtonian potential $\Phi(r) \sim mM/M_p^2$: **Finite potential!**
- **Frolov**: "A finite solution is not neccessary regular one"; reproduces solution.

Comments

Evaluation

The result $\Phi(r)$ is nothing new! The same linearized gravity solution with

$$\mathtt{erf}\left(\frac{rM}{2}\right) = \gamma\left({}^1/_2; \left(\frac{rM}{2}\right)^2\right) \equiv \gamma({}^1/_2; {}^{r^2}/_{4\theta})$$

has already been done by Nicolini 2005 (arXiv:hep-th/0507266), where $\theta = 1/M$ is the NCBH fundamental length and therefore M is the minimal length. The same result was also recovered in 2005 by Gruppuso (arXiv:hep-th/0502144) with the `erf` representation. None of them were cited by Biswas, Gerwick, Koivisto, Mazumdar 2011 (arXiv:1110.5249).

Boosting the solution

- Object of investigation: Collapsing spherical thin null shell.
- To do so: Boost point source along y-axis with constant ultrarelativistic velocity β. After comoving transformation $(t, y) \to (v, u)$, the field is found as

$$\Phi = -4GM \, F(\zeta_\perp^2) \, \delta(u)$$

$$F(z) = \int_0^\infty \frac{ds}{s} f(s) \, e^{-z/(4s)}$$

- Remember: Only HD/GF gravity has $f \neq 0$. For example in GF gravity,

$$F(z) \sim z - \frac{1}{4} z^2 + O(z^3) \qquad (1)$$

Apparent horizon or not

- Author considers ominous set of photons added to the spacetime: $ds^2 = ds_0^2 + \langle dh^2 \rangle$
- Determine the position of an apparent horizon by

$$0 = (\nabla g_{\theta\theta})^2 = (\nabla (r^2 - \frac{GM}{r} z F(z)))^2$$

- Find: For small enough mass M there is no apparent horizon! $\rightarrow$ Mass gap for mini-black hole production.
- Funnily: "The divergence is a result of the unphysical assumption, that the thickness of the null shell is zero, and for the collapse of the shell with finite thickness this divergence becomes softer or is absent."

Evaluation

In comparison with our models, eg. GUP, holographic/ self-regular principles or NCBHs, Frolov's result is probably not suprising: He spent a great effort to discover the G-lump which is smaller than the minimal mass and has no event horizon. Such an example is displayed for NCBH-like metric components g_{00} in the lower plot (blue line $\hat{=}$ "mass gap" gravitational potential).